Journal of Coastal Research	SI 64	pg - pg	ICS2011 (Proceedings)	Poland	ISSN 0749-0208

Nearshore Dynamics of Tsunami-like Undular Bores using a Fully Nonlinear Boussinesq Model

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ABSTRACT

Tissier, M., Bonneton, P., Marche, F., Chazel, F. And Lannes, D., 2011. Nearshore dynamics of tsunami-like undular bore using a fully nonlinear Boussinesq model. Journal of Coastal Research, SI 64 (Proceedings of the 11th International Coastal Symposium), pg – pg. Szczecin, Poland, ISSN 0749-0208

When tsunami wave fronts reach the shore, they can evolve into a large range of bore types, from undular non-breaking bore to purely breaking bore. It is the complex competition between non-linearities, dispersive effects and energy dissipation which will govern their transformations, making the prediction of their evolution a challenging task for numerical models. In this paper we investigate the ability of a fully nonlinear Boussinesq model, SURF-WB, to predict bore dynamics in a large range of Froude numbers. The model is first applied to the formation of undular bores, and compared with laboratory data. Its ability to predict the different bore shapes is then investigated. Finally, the effects of the bore transformation on wave run-up over a sloping beach are considered.

ADDITIONAL INDEX WORDS: *Undular bore, hydraulic jump, Froude number, run-up, dispersive effects, Green-Naghdi equations.*

INTRODUCTION

Tsunamis propagating in the open ocean are basically non dispersive long waves. However, the integration of weak dispersive effects can become significant when the propagation takes place over a long time, and end up modifying the tsunami waves. Dispersive effects finally become significant in shallow-water regions (see Dalrymple *et al.* 2006, Løvholt *et al.* 2008, Grue *et al.* 2008). In the nearshore, the competition between non-linearities, dispersive effects and energy dissipation will govern the transformation of tsunami wave fronts. They can evolve into a large range of bore types, from undular non-breaking bore to purely breaking bore. For instance, during the 1983 Japan Sea tsunami, tsunami-made waves ascended different rivers, evolving into breaking bores either followed or not by an undulated wave train (Tsuji *et al*., 1990). The propagation of tsunami waves in river channels was also recently studied by Yasuda (2010), who applied his Boussinesq-type model to undular bores resulting from the September 2003 Tokachi-oki tsunami. The natural disaster caused by the 26 December 2004 Indian Ocean Tsunami has led to numerous studies, with emphasis on the potential generation of disastrous flood waves. During this event, undular bores were observed in the Strait of Malacca. Video recordings showing the run-up of 15 to 20s period waves associated with this undular bore motivated a recent study by Grue *et al.* (2008). They studied numerically how the initial tsunami wave can turn into an undular bore while propagating along this shallow strait. They showed that the formation of undulations leads to a significant increase of surface elevations, and that all the individual elevation waves will eventually develop into solitary waves at the end. However, wave breaking was not taken into account in these studies and should significantly affect the transformation of the undular bore while propagating shoreward. The prediction of bore transformations in the nearshore is therefore a difficult task, but it is an essential step for a better understanding of tsunami run-up and impact on coastal structures.

Laboratory experiments (among others Favre, 1935, Treske, 1994, Chanson 2009) showed that the wave Froude number Fr controls the bore shape. In the supercritical regime (Fr>≈1.4), the bore consists of a steep front, while undulations are growing at the bore head in the near-critical state (Fr≈1). However the transition between these states is still poorly understood, and can become very complex for coastal applications with evolving bathymetries. As it involves the complex interaction between nonlinear, dispersive and dissipation effects, the description of the different bore types is a very challenging test case for numerical models. To our knowledge, previous numerical studies concerning bore dynamics using depth-averaged models were devoted to either purely breaking bores, or non-breaking undular bores. Breaking hydraulic bores are traditionally solved using shock-capturing models based on nonlinear shallow-water equations (see Brocchini and Dodd, 2008), while test cases concerning undular bores have been used to validate several Boussinesq-type models (*e.g.* Wei *et al.*, 1995, Soares-Frazao and Zech, 2008, Grue *et al*., 2008, Madsen *et al*., 2008). Recently, Mignot *et al.* (2009) investigated the ability of a fully-nonlinear Boussinesq model to describe river flows including shocks. They showed that their model was able to reproduce the overall wave front dynamics, but highlighted some potential drawbacks of their viscous-like breaking model for riverine applications including shocks.

In this study, we assess the ability of the 1D Fully Nonlinear Boussinesq model introduced in Bonneton *et al.* (2010a,b) and Tissier *et al.* (2010) to predict bore dynamics in a large range of Froude numbers. This model has been developed as an extension of the shock-capturing shallow-water model SURF-WB (Marche *et al.*, 2007), using hybrid finite-volume finite-difference schemes. The numerical strategy was developed such as it preserves the initial properties of SURF-WB concerning its natural treatment of wave breaking (see Bonneton *et al.*, 2010a). The model is briefly described in the first section. Undular bore development and evolution are studied in the next section, using experimental data. We then study the transition from one type of bore to the other, and evaluate the hydrodynamical conditions that control the transition for idealized cases. Finally, the run-up of an undular bore over a sloping beach is considered and compared to the predictions given by a nonlinear shallow water model.

NUMERICAL MODEL

We briefly present in this section the main characteristics of our numerical model. The reader is referred to Bonneton *et al*. (2010a) and Chazel *et al*. (2010) for further details.

Governing equations

The model is based on the Serre Green-Naghdi (S-GN) equations, considered as the basic fully nonlinear weakly dispersive equations (Lannes and Bonneton, 2009). These equations can be formulated in terms of the conservative variables (h,h**v**) in the following nondimensionalized form (see Bonneton et al., 2010a):

$$\begin{cases} \partial_t h + \varepsilon \nabla \cdot (h\mathbf{v}) = 0 \\ \partial_t (h\mathbf{v}) + gh\nabla\zeta + \varepsilon\nabla\cdot(h\mathbf{v}\otimes\mathbf{v}) = -D, \end{cases} \tag{1}$$

where ζ is the surface elevation, $h=1+\varepsilon\zeta-b$ the water depth, b the variation of the bottom topography and $\mathbf{v}=(u,v)$ the depth averaged velocity. $\varepsilon=H/h_0$ is the non-linear parameter with H the wave height and h_0 the typical water depth. ***D*** characterizes non-hydrostatic and dispersive effects, and has been written in a way that it does not require the computation of any third order derivative. It is noteworthy that if ***D***=0, we obtain the Nonlinear Shallow Water Equations (NSWE) in their conservative form. The formulation is therefore well-suited for a splitting approach separating the hyperbolic (NSW) and dispersive parts of the equations, allowing for an easy coupling of the sets of equations. The strategy for wave breaking is based on the following idea: breaking waves will be described locally as shocks with the NSW equations, whereas non-breaking waves will be described with the S-GN equations.

Numerical strategy

At each time step dt, we decompose the solution operator S(•) associated to the equations (1) by the following second order splitting scheme:

$$S(dt) = S1(dt/2)\, S2(dt)\, S1(dt/2), \tag{2}$$

where S1 and S2 are respectively associated to the hyperbolic and dispersive parts of the S-GN equations. S1 is treated using a shock-capturing finite-volume method, while S2 is treated using a classical finite-difference method.

The numerical methods used in S1 are those developed in the well-validated NSW code SURF-WB (Marche *et al.*, 2007, Berthon and Marche, 2008). These schemes are high-order positive preserving well-balanced shock-capturing schemes. They are able to handle breaking bore propagation without any parameterization of the energy dissipation (breaking waves are described as shocks by the model), as well as moving shorelines without any tracking method.

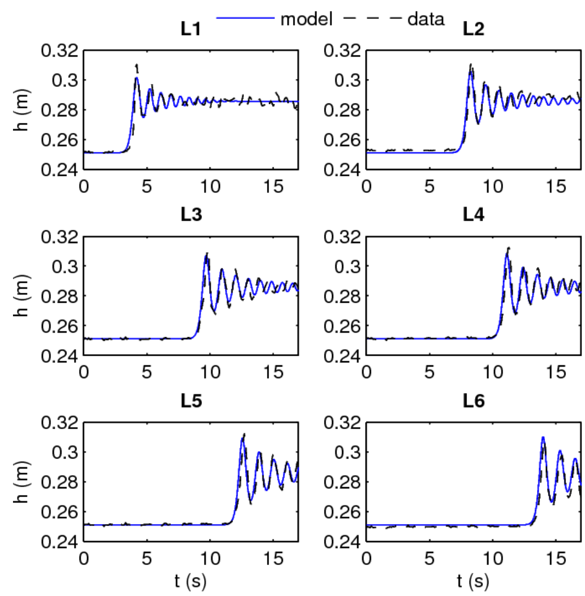

Figure 1. Water level evolution as a function of time at 6 gauges. Comparisons between experimental data (dashed lines) from Soares-Frazao and Zech (2002) and model prediction (plain lines). Fr=1.104.

Wave breaking

By skipping locally the dispersive step when the wave is ready to break, we switch from GN equations to NSWE for a given wave front. Energy dissipation due to wave breaking is then predicted by the shock theory, and no complex ad-hoc parameterization is required.

More precisely, the first step S1 is used to determine where to suppress the following dispersive step (S2) at each time step. We compute the local energy dissipation *Di(x,t)* during S1, and integrate it over each front. After normalization by the theoretical dissipation across the shock, this parameter can be used to characterize the wave fronts, since $\int_{front} D_i(x,t)dx\,/D_{th}$ is close to one for fully broken waves, and close to zero for non-breaking waves. We also compute the front slope Φ at the end of S1, and define two angles, Φi and Φt, corresponding to the initiation and termination of breaking (Schäffer *et al.*, 1993). We choose Φi=30° and Φt=8°, which are the optimal angles determined by Cienfuegos *et al.* 2010 for their S-GN model. Finally, the following method is applied in order to decide if we switch for a given wave front from one set of equations to the other (Tissier *et al.*, 2010). If $\int_{front} D_i(x,t)dx/D_{th} > 0.5$ and Φ>Φf: the wave is broken and will keep breaking (as long as Φ>Φf). The wave front is locally governed by NSWE. If $\int_{front} D_i(x,t)dx/D_{th} \leq 0.5$ and Φ<Φi: the wave is not breaking. The wave is governed by S-GN equations. The switch to NSWE will eventually occur if Φ gets larger than Φi.

This model was successfully applied to the shoaling and breaking of periodic waves on sloping beaches, which correspond to breaking bores with high values of the Froude number. The model was also validated for the description of shoreline motions, showing a very good agreement with laboratory data. In the next

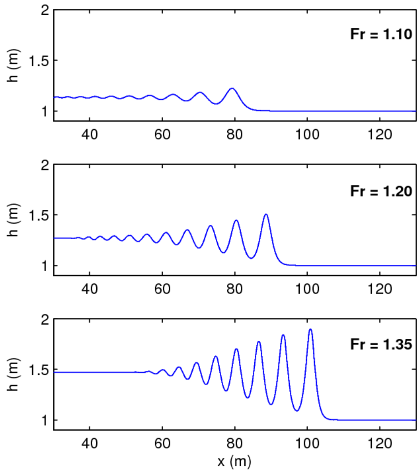

Figure 2. Free surface profiles at t=24s of hydraulic bores with Froude numbers varying from 1.10 to 1.35.

section, we assess the ability of our model to describe bore dynamics for moderate Froude numbers (close to unity).

FROM UNDULAR TO BREAKING HYDRAULIC BORE

Development of an Undulated Wave Train

Pioneering experimental works concerning the development of undular bores were performed by Favre (1935), who studied their formation due to a rapid opening or closure of a gate. Similar experiments were conducted by Soares-Frazão and Zech (2002), and studied numerically in Soares-Frazão and Guinot (2008). The experimental data are used here to validate our S-GN model. For this experiment, we consider a channel initially at rest with an initial water height h_0=0.251m. At t=0s, we impose a discharge at the gate Q_0=0.06$m^3.s^{-1}$. The Froude number of the resulting bore is Fr=1.104. For the simulations, the grid size of the mesh is dx=0.2m and we consider a Courant number of 0.9. Figure 1 compares experimental and numerical time-series of water elevation at the wave gauges. It shows that the model predicts accurately the bore celerity, and that the agreement is good in term of amplitude and wavelength of the secondary waves. It can be noticed that the growth of the first wave is faster in the experiments than predicted by our numerical model. Similar results were obtained by Soares-Frazão and Guinot (2008) with their Boussinesq model.

Transition from Undular to Purely Breaking Bore

We consider in this section the transformations of an initial step over a flat bottom defined by:

$$\begin{cases} h(x,0) = \dfrac{1}{2}(d_b - d_a)(1 - \tanh(x/a)) + d_a, \\ v(x,0) = \dfrac{1}{2}(u_b - u_a)(1 - \tanh(x/a)) + u_{,a}, \end{cases} \qquad (3)$$

for Froude numbers varying from 1.10 to 1.90. d_a and d_b are the water depth in front and behind the bore, u_a and u_b the corresponding depth-averaged velocities. We set u_a=0 (channel initially at rest), d_a=1m and a=2m. For each Froude number, we then compute d_b and u_b by solving the mass and momentum conservation conditions across the bore. Figure 2 and 3 show the bore shapes at t=24s for the different Froude numbers.

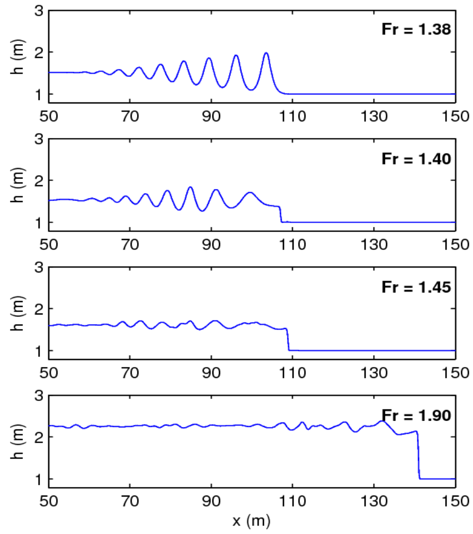

Figure 3. Free surface profiles at t=24s of hydraulic bores with Froude numbers varying from 1.38 to 1.90.

For Fr<1.40, the initial step evolves into an undular jump. We observe that the wavelengths of the secondary waves decrease with increasing Froude number, while their amplitudes increase (see Figure 2). This is a well-known result which has been for instance presented in Treske (1994) or Chanson (2009). For Fr=1.40, a wave train is again formed but its face is broken. Although the first wave seems too damped in comparison with experimental results, the overall shape is qualitatively well-reproduced (see for instance the bore picture at Fr=1.35 from Treske (1994), or the longitudinal profiles from Binnie and Orkney (1955)). For higher Froude numbers, we obtain a purely breaking bore. It can be observed that some disturbances occur behind the breaking fronts. Their physical relevance still needs to be explored.

A review of the recent experimental results concerning hydraulic bores is given in Chanson (2009). He observed that the wave amplitude data showed a local maximum value for Fr=1.27 to 1.7, depending upon experimental conditions such as the channel width, and that the undulated wave train was disappearing for Fr=1.5 to 3. Our model predicts that the maximum amplitude of the secondary waves is obtained for Fr=1.38, and the transition to a purely breaking bore starts for Fr=1.40, which is therefore in good agreement with experimental data.

RUN-UP OF AN UNDULAR BORE OVER A PLANAR BEACH

In the previous sections we have shown that our S-GN model is able to reproduce accurately the main features of different bore types. Hereafter, we consider the effects on wave run-up of the transformation of an initial soliton into an undular bore. In order to quantify the importance of dispersive effects for these transformations, we also compute the evolution of the same initial wave without considering the dispersive effects, *i.e.* solving only the NSW part of the equations (same code but considering ***D***=0, see Equation (1)). The computations without dispersive effects are represented by dashed-lines in Figures 4 to 6.

Madsen *et al.* (2008) studied how a long wave can disintegrate into an undular bore close to the beach. In order to obtain a realistic undulated incoming wave for our run-up simulation, we first perform a computation similar to the one presented in Madsen

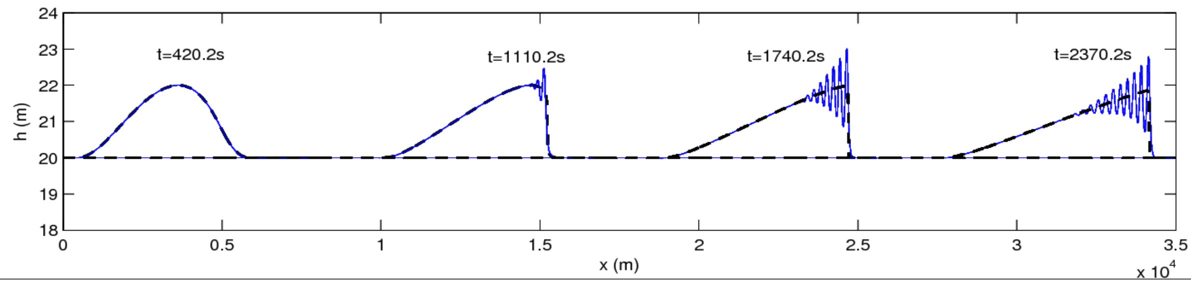

Figure 4. Disintegration of a long wave (a_0= 2m, T=780s) propagating over a flat bottom with h_0=20m. Plain lines: computed with our S-GN model. Dashed-lines: NSW model.

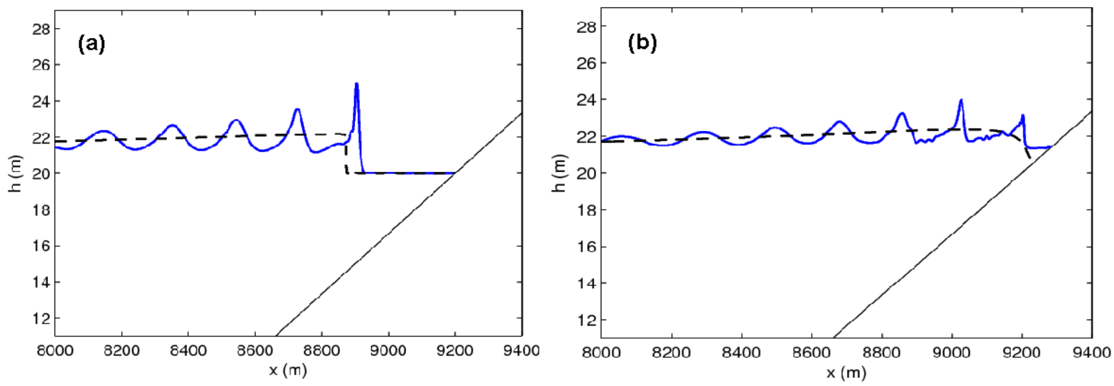

Figure 5. Propagation of the undular bore over the sloping beach (a) At t=t0+124.25s; (b) At t=t0+166.25s. Plain lines: S-GN model. Dashed-lines: NSW model.

et al. (2008). We consider the evolution of a long wave reaching a shallow flat region where non-linearity is significant. The input wave is a $\sin^2$ wave with amplitude a_0=2.0m and period T=780s propagating over a constant depth h_0=20m. The wave period was chosen to match the observations during the 2004 Indian Ocean tsunami.

The wave profiles at different times are represented in Figure 4, with (plain lines) and without dispersive effects (dashed lines). The S-GN model predicts the formation of an undulated wave train, which starts developing at the wave front. After propagating 2370.2s, the resulting wave is about 1.5 times higher than the one predicted by the NSW model. The individual elevation waves could eventually evolve into solitary waves if they were propagating long enough in shallow water, but, according to Madsen *et al.* (2008), it rarely happens because of geophysical constraints. This case is therefore not considered in this study, and we use the wave profiles at t=2370.2s (see Figure 4) as the initial conditions for the run-up simulations (with and without dispersive effects).

At t=0, the undular bore is located at the toe of a 1:60 sloping beach. For the simulation, the grid size of the mesh is dx=1m and the time step dt=0.07s. Figure 5 shows the transformation of the bore as it propagates shoreward, during shoaling (a), and breaking (b). The wave height predicted with the S-GN model just before breaking (see Figure 5b) is about 2.3 times the wave height obtained without dispersive effects. Figure 6 compares the time-evolution of the run-up predicted by the S-GN code and the NSW code. When we do not account for the dispersive effects (no secondary waves develop), the run-up is underestimated of only 11%. It seems to confirm what has been hypothesized by Madsen *et al.* (2008), that is to say that the development of shorter waves riding on the top of a tsunami may not impact significantly the run-up caused by the main tsunami. However, the development of the secondary waves and the subsequent elevation raise of the main front will certainly have a local effect on tsunami wave impact on coastal structures, and is therefore an important feature.

CONCLUSIONS

In this paper, we investigated the ability of the fully nonlinear Boussinesq model SURF-WB (Bonneton *et al.* (2010a,b), Chazel *et al.* (2010) and Tissier *et al.* (2010)) to predict bore dynamics for a large range of Froude numbers. Promising results were found concerning the ability of our model to reproduce the transition from purely undular bore to strongly breaking bore. Despite our highly idealized simulations (no friction effects, cross-shore model) the characteristic Froude number for the transition (Fr=1.40) was close to the typical values obtained during laboratory experiments (*e.g.* Treske, 1994). To our knowledge, SURF-WB is the first Boussinesq model able to reproduce this transition, which is a challenging task since it involves the complex interaction between nonlinearities, dispersive effects and energy dissipation.

Finally, the transformation of an undulated wave while propagating over a 1:60 sloping beach was considered. For this simulation, the maximum wave height predicted with our S-GN model was up to 2.3 times the one predicted by the NSW part of the code only, *i.e.* without accounting for the dispersive terms of the equations. The run-up was computed in both cases and compared. We showed that the main characteristics of the run-up were well-reproduced by the NSW code only, with an underestimation of the maximum run-up of about 11%. The run-up is therefore mainly determined by the main tsunami wave and not by the short waves riding on the top of the tsunami. This result is in agreement with Madsen *et al.* (2008), who suggested that the effect on wave run-up of the disintegration of tsunami waves while propagating shoreward was often overestimated. However, the modeling of this transformation remains important, since the formation of secondary waves, and the subsequent increase of the tsunami front height, will certainly have an a significant impact on coastal structures.

Recent intensive high temporal and spatial resolution field works on the dynamics of tidal bores (Bonneton *et al*., 2011) will give us a new opportunity to validate our model, since tidal bores exhibit similar behaviors as tsunami waves propagating in shallow-water estuaries.

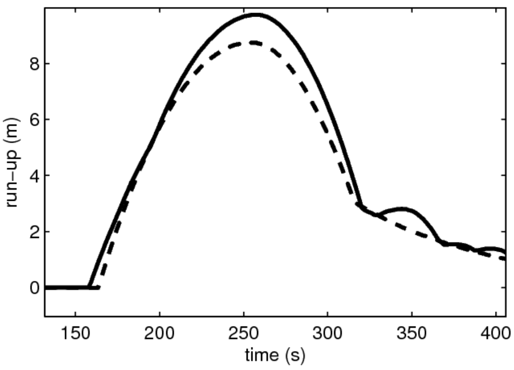

Figure 6. Run-up of the undular tsunami on the sloping beach (plain line) computed by our S-GN model. Dashed-line: same simulation without dispersive effects.

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ACKNOWLEDGEMENT

This work has been supported by the ANR MathOcean. The Ph.D. thesis work of M. Tissier is funded by MISEEVA (ANR). The authors are grateful to S. Soares-Frazão for providing the experimental data. The authors acknowledge V. Marieu for his helpful assistance.