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Turbulence-mean field interactions and layer formation in a stratified fluid

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Abstract – We briefly review some recent numerical and theoretical results on turbulence-mean field interactions in stratified turbulent flows. Results from direct numerical simulations are presented. In these simulations, turbulence decay in a stably-stratified fluid is investigated with a pseudo-spectral numerical code solving the fully non-linear Navier–Stokes equations under the Boussinesq approximation with periodic boundary conditions. The flow is decomposed into a turbulent field and a horizontal mean flow $\overline{\mathbf{u}}(z,t)$ defined as the average of the horizontal velocity component in a horizontal plane at height z and time t. Similarly, the density field is decomposed into a turbulent field and a (stable) mean density profile $\overline{\rho}(z,t)$ defined as the average of the density field in a horizontal plane at height z and time t. Attention is paid to the effect of the turbulent velocity field on an initial z-periodic horizontal mean flow or an initial z-periodic perturbation of the mean density profile. The results show that the turbulence-mean field interactions are strongly affected by the buoyancy forces: when a strong stratification is applied, the perurbations in the mean profiles tend to grow, which accounts for the tendency of stratified turbulence to form horizontal layers.

The linear processes involved in these turbulence-mean field interactions are briefly discussed using a slightly non-homogeneous version of the Rapid Distortion Theory. The results of the linear model show that in the first stage of decay of turbulence, the eddy viscosity and diffusivity take negative values when the flow is subject to a strong stable stratifcation. These conclusions are in good agreement with the results from direct numerical simulation for short time.

We conclude that the linear processes play a significant role in these turbulence-mean field interactions and are widely involved in the formation of horizontal layers in stratified geofluids such as oceans and atmospheres. © 2001 Éditions scientifiques et médicales Elsevier SAS

1. Introduction

The formation of horizontal layers in stratified turbulent flows was first explained in general terms by Phillips [1] who asked "Turbulence in a strongly stratified fluid-is it unstable?". Later, Posmentier [2], and independently Puttock [3], proposed a simple mechanism for the formation of these layers, which in oceanography are called salinity finestructures. Let us consider the equation for conservation of the mean salinity $S(z,t)$ (where z is the vertical coordinate and t is time) in a horizontally homogeneous but vertically varying profile,

$$\partial_t S = -\partial_z F, \tag{1}$$

where F is the flux associated with turbulent fluctuations and microscale mixing. Posmentier [2] pointed out that equation (1) may be written as:

$$\partial_t S = -F^\star \partial_{zz} S, \tag{2}$$

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where $\star$ denotes differentiation with respect to $\partial_z S$ and $F^\star$ has dimension of a diffusivity. The sign of $F^\star$ is a key point which determines the stability of the solutions for S. Equation (2) has stable solutions if $F^\star < 0$ and unstable solutions if $F^\star > 0$. In the unstable case, any perturbation in the mean density profile is amplified leading to the formation of layers, however the problem is mathematically ill-posed since the growth rate diverges for small-scale perturbations. Furthermore, this theoretical discussion did not differentiate between stratified turbulent flows with and without mean shear, and did not account for the energy supply to the flow.

Several phenomenological models (see for instance Barenblatt et al. [4] and Balmforth et al. [5]) have been used to estimate the buoyancy fluxes as a function of the mean density gradient in order to simulate layering processes. Various physical arguments, such as the existence of a finite mixing length or a finite adjustment time of turbulence, have been included in these models in order to avoid singularities in the solutions and to predict the layer formation. In these models, the turbulence is assumed to be in a state that is quasi-steady, and developing only as slowly as the mean gradients. This is consistent with the conditions of the stirred tank experiment of Park et al. [6]. There are other situations where the turbulence is changing rapidly, for example decaying, and then the model (2) is not necessarily applicable. At the early stages of homogeneous stratified grid turbulence (Rottman and Britter [7]), no maximum in the curved of the buoyancy flux against the mean density gradient and no layering were observed. On the other hand, through different mechanisms to those proposed by Phillips [1], layers were observed in the final stage of decay of turbulence by Pearson and Linden [8], who developed a theory where viscous rather than turbulent shear stresses balanced the buoyancy forces.

The problem of ‘layering’ in stratified turbulence has also been addressed theoretically by Godeferd and Cambon [9], who studied in details the energy transfers between the various components of the flow field. From this point-of-view, layering is associated with the anisotropic properties of turbulence, which is a different mechanism from those involving the growth of the mean shear and density profiles.

The case where the fluctuation field has small amplitude compared to the variations in the mean fields has been extensively addressed in the past using a linear stability analysis (e.g., Miles [10], Howard [11] and more recently Majda and Shefter [12]). Here the turbulence-mean field interaction is simulated in the more realistic situation where the amplitude of the variations in the mean fields are of the same order as the amplitude of the turbulent field. To describe this turbulence-mean interaction in the first stage of decay of turbulence, direct numerical simulation is a powerful tool which allows us to describe accurately the short-time evolution of the stratification and mean flow profiles.

In the present paper we present some numerical simulations which highlight the effect of a stable stratification on the turbulence-mean field interactions, where the mean field is either the mean fluid density profile or the mean horizontal velocity profile (section 2). In section 3, a slighlty non-homogeneous version of the Rapid Distortion Theory is briefly presented to discuss the linear processes involved in these interactions. Our conclusions are provided in section 4. Details on the numerical simulations and the rapid distortion model may be found in Galmiche et al. [13] and Galmiche and Hunt [14] respectively.

2. Results from numerical simulations

In the past few years, a number of direct numerical simulations (e.g., Riley et al. [15], Métais and Herring [16], Gerz and Schumann [17], Kimura and Herring [18], Galmiche et al. [13]) of freely decaying turbulence have improved our understanding of momentum and buoyancy diffusion and layer formation in stratified turbulence.

Let $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be a Cartesian frame where $\mathbf{e}_3$ is antiparallel to the gravity g. The spatial coordinates will be denoted by $\mathbf{x} = (x, y, z) = (x_1, x_2, x_3)$, the velocity field by $\mathbf{u}(\mathbf{x}, t) = (u, v, w) = (u_1, u_2, u_3)$ and the density field by $\rho(\mathbf{x}, t)$. In order to facilitate the analysis, we shall make the Boussinesq approximation and consider the case of periodic boundary conditions in the three directions (with periodicity L), such as those imposed in most direct numerical simulations (e.g., Galmiche et al. [13]). The Brunt–Vaïsälä frequency associated with the background stratification will be denoted by N_0.

To address the question of layer formation, it is useful to decompose the density field into a turbulent field and a (stable) mean density profile $\overline{\rho}(z, t)$ defined by:

$$\overline{\rho}(z, t) = \langle \rho(\mathbf{x}, t) \rangle^{xy} = \frac{1}{L^2} \int_0^L \int_0^L \rho(\mathbf{x}, t) \, \mathrm{d}x \, \mathrm{d}y. \tag{3}$$

Similarly, the velocity field may be decomposed into a turbulent field and a horizontal mean flow $\overline{\mathbf{u}}(z, t)$ defined as the average of the horizontal velocity component in each horizontal plane at height z and time t:

$$\overline{\mathbf{u}}(z, t) = \langle u(\mathbf{x}, t) \rangle^{xy} = \frac{1}{L^2} \int_0^L \int_0^L u(\mathbf{x}, t) \, \mathrm{d}x \, \mathrm{d}y. \tag{4}$$

In models of stratified geophysical flows such as oceanic or atmospheric flows, one of the main difficulties is generally to model the effect of turbulence on the mean quantities $\overline{\mathbf{u}}$ and $\overline{\rho}$, especially when the energy is transferred from small to large scales. Neglecting viscous dissipation, the time evolution equation for $\overline{u}(z, t)$ becomes:

$$\partial_t \overline{u} = -\partial_z F_u, \tag{5}$$

where $F_u = \langle uw \rangle^{xy}$ is the vertical turbulent momentum flux. Similarly, the equation for $\overline{\rho}(z, t)$ reads:

$$\partial_t \overline{\rho} = -\partial_z F_\rho, \tag{6}$$

where $F_\rho = \langle \rho w \rangle^{xy}$ is the vertical turbulent buoyancy flux.

In the absence of a stratification (i.e. when ρ is a passive scalar), the eddy fluxes of mass and momentum are generally modelled using the concept of eddy diffusivity and viscosity which depend on local properties such as the mixing length and intensity of turbulence. In most cases, these eddy coefficients have positive values. When a stable stratification is present, the buoyancy term does not appear explicitly in equations (6) and (5), but the stratification affects the fluctuations of the flow field and thus the turbulent fluxes. The mechanisms affecting the transport of mass and momentum in a stratified fluid are partly due to the wavy properties of the medium which make allowance for the propagation, distortion and interactions of internal gravity waves. This has an effect on the correlations $\langle uw \rangle^{xy}$ and $\langle \rho w \rangle^{xy}$ and largely modifies the concepts of eddy viscosity and diffusivity.

A number of numerical simulations (Riley et al. [15], Gerz et al. [19]) and results from the Rapid Distortion Theory (Hanazaki and Hunt [20]) show that in decaying, stratified turbulent flows with homogeneous and isotropic initial conditions, the buoyancy fluxes tend to oscillate and eventually become counter-gradient. When a vertical mean shear is present, the direct numerical simulations of Gerz et al. [19] show that the buoyancy and momentum turbulent fluxes both oscillate and eventually become counter-gradient. However, when the mean shear and stratification are both uniform, the fluxes do not depend on height z and it is clear from equations (6) and (5) that the mean profiles $\overline{\rho}$ and $\overline{u}$ remain unaffected as the turbulence evolves. Nevertheless, following Phillips [1] and Posmentier [2], it is probable that for a sufficiently large stratification, an initial perturbation in the mean density profile may grow under the effect of the counter-gradient buoyancy fluxes, leading to the formation of horizontal layers. Similarly, one may expect that shear layers can develop under the effect of the counter-gradient momentum fluxes when the turbulent field is perturbed by a non-uniform mean shear profile. One consequence of these phenomena is that the concepts of positive eddy viscosity and diffusivity fail and may be replaced by negative coefficients.

Direct numerical simulations have been performed by Galmiche et al. [13] to investigate the behaviour of these eddy coefficients in stratified turbulence. The flow is simulated with a pseudo-spectral numerical code solving the fully non-linear Navier–Stokes equations under the Boussinesq approximation with periodic boundary conditions. In these simulations, an initially homogeneous and isotropic turbulent velocity field is left to decay without any external forcing in the presence of a background stratification, and a z-periodic pertubation is introduced initially either in the horizontal mean flow profile (Simulation A) or in the mean density profile (Simulation B). The initial mean profiles ($t = t_0$) are

$$\overline{u}(z, t = t_0) = \overline{u}_0 \cos(2\pi z/L) \quad \text{and} \quad \overline{\rho}(z, t = t_0) = \overline{\rho}_l(z) \quad \text{in Simulation A} \tag{7}$$

and

$$\overline{u}(z, t = t_0) = 0 \quad \text{and} \quad \overline{\rho}(z, t = t_0) = \overline{\rho}_l(z) + \overline{\rho}_0 \cos(2\pi z/L) \quad \text{in Simulation B,} \tag{8}$$

where L is the size of the periodic domain and $\overline{\rho}_l(z)$ is the linear density profile associated with the background stratification. The amplitude $\overline{u}_0$ in Simulation A (respectively $\overline{\rho}_0$ in Simulation B) is such that the initial energy $E_{\overline{u}}(t_0)$ of the mean flow profile (respectively the initial energy $E_{\overline{\rho}}(t_0)$ of the mean density profile) is one third of the initial total energy (kinetic plus potential) of the turbulent flow $E(t_0)$. The mean profiles are then left to evolve under the effect of the eddy turbulent fluxes. In these simulations, the initial Reynolds number is $Re = u'_0 l_0/\nu = 55$ (where l_0 and u'_0 are the integral lengthscale and the r.m.s. velocity of the initial turbulent field respectively, and ν is kinematic viscosity) and the Prandtl number is taken equal to unity. The intensity of the stratification may be characterized by the value of the Froude number $Fr = u'_0/N_0 l_0$. In both simulations A and B, three cases are considered: $Fr = 0.12$ (strongly-stratified), $Fr = 1.2$ (moderately-stratified) and $Fr = \infty$ (non-stratified). In the non-stratified simulations, the density field has to be seen as a passive scalar getting mixed by the turbulent motions. Simulations A and B are denoted by ASS, AMS, ANS, BSS, BMS and BNS in the strongly-stratified, moderately-stratified and non-stratified cases respectively.

In order to trace back the effect of the stratification on the temporal behaviour of the perturbations, the evolution of $E_{\overline{u}}(t)$ in Simulations ASS, AMS and ANS is plotted on *figure 1* and the evolution of $E_{\overline{\rho}}(t)$ in Simulations BSS, BMS and BNS is plotted on *figure 2*. We have also plotted the analytical solutions for $E_{\overline{u}}(t)$ and $E_{\overline{\rho}}(t)$ when the effect of turbulence is ignored (purely viscous and diffusive decay of the mean profiles). On all these plots, the timescale is $\tau_0 = l_0/u'_0$, the initial turnover timescale of turbulence.

In Simulation A with moderate stratification (Simulation AMS), the turbulent diffusion of momentum remains efficient during one or two turnover timescales and $E_{\overline{u}}$ decays like in the non-stratified experiment ANS (see *figure 1*). After two or three turnover timescales, the effect of the restoring buoyancy forces causes the fluid particles to reduce their vertical motion which affects the turbulent stresses and slows down the mean flow decay. After four or five turnover timescales, the turbulence becomes almost inefficient in affecting the mean flow and the rate of decay of $E_{\overline{u}}$ tends to the viscous rate. In the strongly-stratified simulation (ASS), the effect of the buoyancy forces on the mean flow is more dramatic. In the very early stages of decay, the evolution of $E_{\overline{u}}$ is close to the decay observed in the non-stratified simulation (ANS), but the mean flow is affected by the stratification as soon as $t - t_0 \simeq 0.2\tau_0$ ($\simeq 1.7N^{-1}$). At this time, $E_{\overline{u}}$ increases and not only starts oscillating but also keeps increasing permanently during two or three turnover timescales. $E_{\overline{u}}$ becomes greater than the viscous solution when $t - t_0 \simeq 0.35\tau_0$ ($\simeq 3N^{-1}$). After four turnover timescales, $E_{\overline{u}}$ decays at the viscous rate

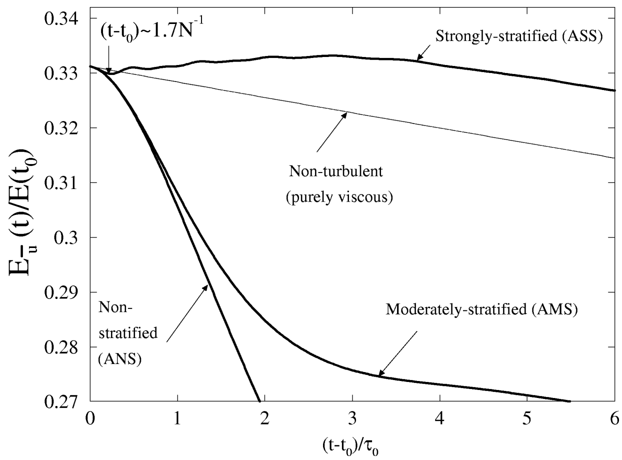

Figure 1. Evolution of $E_{\overline{u}}$, the energy of the mean flow profile in simulations ANS, AMS and ASS (Galmiche et al. [13]). The time unit is $\tau_0 = l_0/u'_0$, the turnover timescale of turbulence at $t = t_0$. For comparison, the analytical solution for $E_{\overline{u}}(t)$ has been plotted when the effect of turbulence is ignored (purely viscous decay).

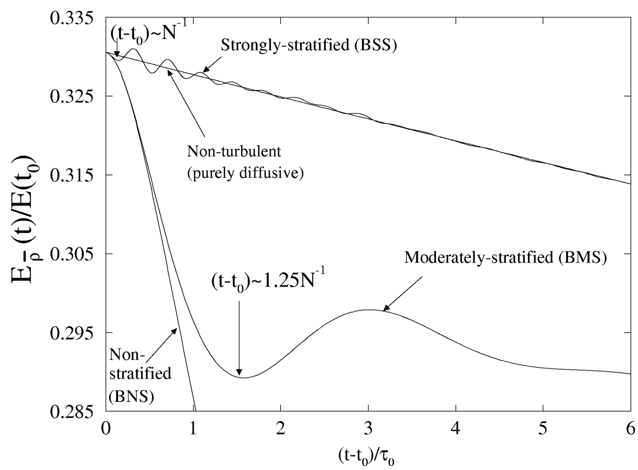

Figure 2. Evolution of $E_{\overline{\rho}}$, the energy of the mean density profile in simulations BNS, BMS and BSS (Galmiche et al. [13]). The time unit is $\tau_0 = l_0/u'_0$, the turnover timescale of turbulence at $t = t_0$. For comparison, the analytical solution for $E_{\overline{\rho}}(t)$ has been plotted when the effect of turbulence is ignored (purely diffusive decay).

but remains larger than the viscous solution. These results show that the buoyancy forces cause the turbulent motions to transfer energy to the mean motion, which induces a net acceleration of the horizontal current.

In Simulation B (*figure 2*), a significant increase and weak amplitude oscillations of the potential energy $E_{\overline{\rho}}$ are observed when a stratification is present (Simulations BMS and BSS) as soon as the fluid particles are subject to the restoring buoyancy forces ($t - t_0 \simeq 1.25N^{-1}$ in Simulation BMS and $t - t_0 \simeq N^{-1}$ in Simulation BSS). The initial perturbation of the mean density profile is thus alternatively damped and amplified, so that $E_{\overline{\rho}}$ is the major remaining component of the energy at the end of the stratified simulations. After six turnover timescales, we have $E_{\overline{\rho}}/E_{\overline{\rho}}(t_0) \simeq 0.95$ in Simulation BSS and $E_{\overline{\rho}}/E_{\overline{\rho}}(t_0) \simeq 0.85$ in Simulation BMS, whereas $E_{\overline{\rho}}/E_{\overline{\rho}}(t_0) \simeq 0.4$ in the non-stratified simulation. The oscillations of $E_{\overline{\rho}}$ are faster in the strongly-stratified simulation (BSS) but their amplitude is larger in the moderately-stratified simulation (BMS). In the strongly-stratified case, $E_{\overline{\rho}}$ becomes greater than the purely-diffusive solution after a period of $\simeq 0.25\tau_0$ ($\simeq 2N^{-1}$) and then oscillates with a mean rate of decay equal to the diffusive rate. Thus, the turbulent vertical fluctuations are rapidly damped by the strong stratification, which reduces dramatically the turbulent vertical mass transport. In the moderately-stratified simulation (BMS), the vertical turbulent motions are damped more slowly so that the turbulent mass transport remains efficient until $(t - t_0) \simeq 1.5\tau_0$ ($\simeq 1.25N^{-1}$). As a consequence, the final value of $E_{\overline{\rho}}$ remains lower than the diffusive solution is spite of the oscillation occuring at $(t - t_0) \simeq 1.25N^{-1}$.

All these results show the dramatic effect of a stratification on the eddy diffusion of mass and momentum. In the presence of a strong stratification, the perturbations in the mean profiles tend to be enhanced in the first stage of decay of turbulence, and remain very persistent as turbulence evolves. This traces back the tendency of strongly-stratified turbulence to form horizontal layers. In the next section, we discuss the linear processes involved in this phenomenum.

3. Linear processes

The short-time behaviour of freely decaying turbulence is of particluar interest as it has some crucial consequences on the subsequent evolution of the flow and helps to better understand the formation of layers in stratified fluids. For short times, i.e. $t \ll \tau_0$ where τ_0 is the initial turnover timescale of turbulence, a theoretical study of the problem can be undertaken considering that the linear processes of distortion are dominant compared to the non-linear energy transfers, which develop over a few turnover timescales.

The Rapid Distortion Theory has been used by Galmiche and Hunt [14] to study the effect of a turbulent field on the time evolution of perturbations in the mean flow and density profiles. In the analytical model, the flow is assumed to be strongly stratified ($Fr = N_0^{-1}/\tau_0 \ll 1$) and the non-linear transfers are neglected for short times ($t \sim N_0^{-1} \ll \tau_0$). Under these assumptions, together with periodic boundary conditons, the linearized equations of motion under the Boussinesq approximation are solved for short times in the Fourier space (see Townsend [21] for instance). The details of the analysis may be found in Galmiche and Hunt [14]. To adress the question of turbulence-mean field interaction, it is necessary to take the spatial and temporal vartiations of the mean fields into account. Here, the mean shear and stratification are assumed to vary slowly with height compared to the lengthscale l_0 associated with the turbulent field. One can calculate:

(i) the short-time evolution of the momentum and buoyancy fluxes at each altitude, as a function of the local mean shear and stratification;

(ii) the feed-back effect of the eddy fluxes on the temporal behaviour of the mean flow and mean density profiles.

This leads to an analytical solution for the short-time evolution of the mean profiles. The results may be interpreted in terms of eddy viscosity $\nu_e(t)$ and diffusivity $\kappa_e(t)$ defined by

$$\partial_t \overline{u} = \big(\nu_e(t) + \nu\big)\partial_{zz}\overline{u} \tag{9}$$

and

$$\partial_t \overline{\rho} = \big(\kappa_e(t) + \kappa\big)\partial_{zz}\overline{\rho}. \tag{10}$$

The time evolution of these coefficients traces back the effect of the turbulent field on the mean fields when they are perturbed initially. Their value is plotted on *figures 3* and *4* as a function of time and is compared to the results from the strongly-stratified numerical simulations of Galmiche et al. [13]. On these plots, ν_e and κ_e are normalized by the molecular viscosity ν and diffusivity κ respectively.

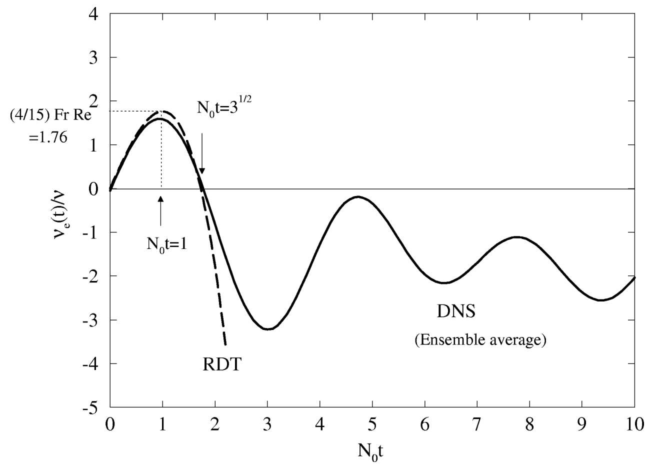

Figure 3. Evolution of the eddy viscosity in the direct numerical simulation of a strongly-stratified turbulent shear flow (Simulation A). The eddy viscosity is normalized by the molecular viscosity and the timescale is the inverse Brunt–Vaïsälä frequency associated with the background stratification. The result is compared with the analytical solution for short times provided by the RDT theory.

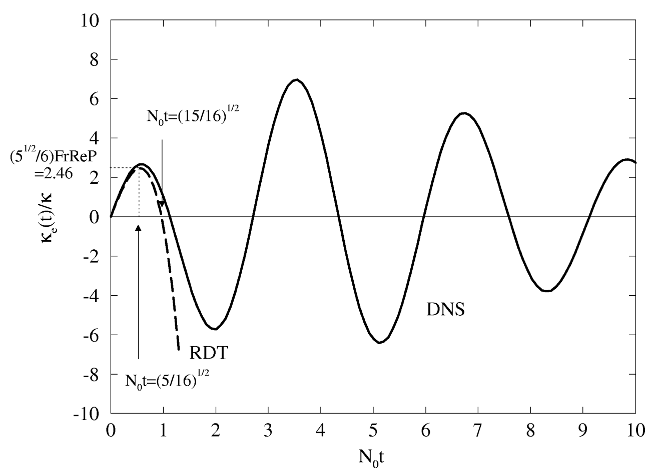

Figure 4. Evolution of the eddy diffusivity in the direct numerical simulation of a strongly-stratified turbulent flow. The eddy diffusivity is normalized by the molecular diffusivity and the timescale is the inverse Brunt–Vaïsälä frequency associated with the background stratification. The result is compared with the analytical solution for short times provided by the RDT theory.

The ratio ν_e/ν is found to reach the maximum value of about 1.7 in the strongly-stratified simulations ASS, and the maximum value is $(4/15) Fr\,Re = 1.76$ in the RDT model (see *figure 3*). Then, the eddy viscosity starts decreasing and becomes negative at $N_0 t \simeq 1.7$ in the DNS and $\sqrt{3}$ in the RDT model.

The ratio κ_e/κ is found to reach a maximum value of $(\sqrt{5}/6) Fr\,Re\,P = 2.46$ in the RDT model (see *figure 4*), which agrees with the DNS (Simulation BSS) results to within 1%. The eddy diffusivity starts decreasing and becomes negative at $N_0 t \simeq 1.1$ in these direct simulations, whereas the value provided by the RDT model is $\sqrt{15/16}$. Physically, this phenomenon may be interpreted as follows: where the density gradient is larger the turbulence is damped and therefore the gradient is locally diffused less by the turbulence than in regions where the gradient is weaker. This is Phillips' mechanism [1] and leads to 'layering' of the vertical density gradient.

The comparison between the RDT solutions and the direct numerical simulations of the fully non-linear equations of motion, show that the turbulence-mean field interactions are largely dominated by the linear mechanisms in the first stage of decay of turbulence, and account for the tendency of strongly-stratified turbulence to develop horizontal layers.

The linear calculations also show that the perturbations in the mean flow and density profiles evolve faster when they have vertical wavelength of order u_0'/N_0 (Galmiche and Hunt [14]). This suggests a theoretical reason why the characteristic thickness of layers is of order u_0'/N_0, as was observed by Park et al. [6]) in laboratory experiments of mixing in salt water. Other quasi-steady state arguments, such as those invoked by Balmforth et al. [5] have also been used to address this question. Of course, this scale is the natural scale for particle displacements and determines density fluctuations measured in the environment (Hunt et al. [22]).

4. Summary and concluding remarks

The results presented in this paper may be summarized as follows:

Direct numerical simulation of freely-decaying, initially homogeneous and isotropic turbulence in the presence of a stable stratification shows that perturbations in the mean shear and mean density profiles tend to grow for short times when the stratification is strong, and remain very peristent as the turbulence evolves. This is an illustration of the property of stratified turbulence to form horizontal layers.

A slightly inhomogeneous version of the Rapid Distortion Theory (Galmiche and Hunt [14]) shows that the effects of a stratification on the wave-mean field interactions can be widely explained by the linear processes of distortion involved in the first stage of decay of turbulence. The analytical model shows that the the eddy viscosity and diffusivity associated with strongly-stratified turbulence become negative at a time of order N_0^{-1} after the beginning of turbulence decay, which accounts for the growth of the perturbations in the mean profiles. This analysis does not require any assumption on the amplitude of the perturbation field compared to the amplitude of the mean profiles but is only based on a comparison of the various timescales of the flow.

It is likely that layering processes in stratified flows involve various mechanisms, such as turbulence-mean field interactions as described in this paper, but also wave-turbulence interactions, wave-wave interactions, wave-mean flow interactions, vortex-vortex interactions and vortex instabilities. Homogeneous, stratified turbulence has been widely investigated by Godeferd and Cambon [9], to show that non-linear energy transfers force the tendency to anistropy and the formation of horizontal structures. This tendency has also been observed in direct numerical simulations (e.g., Métais and Herring [16]) and in recent stratospheric measurements (Alisse and Sidi [23]). The anisotropic features of homogeneous turbulence can be explained by an energy transfer to Fourier modes with mainly vertical wave-vectors, a mechanism which is mainly controled by the vortex-vortex interactions (Godeferd and Cambon [9]). However, it is not clear whether a direct analogy can be made between the modes with quasi-vertical wave vectors and the mean modes (as defined in this paper) which have exactly vertical wave vectors. These modes play a special role in the decomposition of the flow field because they cannot be included neither in the 'wave' part nor in the 'vortical' part of the decompostion. Some work still has to be done to compare and unify the various 'layering mechanisms', e.g., those described in terms of anisotropy (e.g., Godeferd and Cambon [9]), turbulence-mean field interactions (present paper) or vortex pair instabilities (Billant and Chomaz [24]).

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